Extension of Barry/Humblet Model For Limited Wavelength Conversion Model

Miroslav Bahleda

Department of Telecommunication, EF, ŽU v Žiline, Veľký Diel, 010 26 Žilina
e-mail: bahleda@fel.utc.sk, tel: 041 513 2227, fax: 041 513 1520

as a SOCRATES/Erasmus student at

Dresden University of Technology, Communications Laboratory, Chair for Telecommunications, D-01062 Dresden, Germany

Abstract: *This paper deals with the problem of the blocking probability in WDM (wavelength division multiplex) networks. The Barry-Humblet model was proposed for all-optical (fotonic) WDM networks, but only for the uniform traffic load and for the network without and full wavelength conversion. We propose and implement an extension of the Barry-Humblet model for the non-uniform load and also for the limited wavelength conversion with the conversion degree d. This model can be used to determine an optimal place for converters in the sparse full-no and sparse full-limited wavelength conversion networks.*

1 Introduction

The WDM technology is used more and more in the architecture of transport networks due to the many advantages that this technology can offer. It is expected that the use of WDM networks will rapidly grow in the future and it will be the next generation of optical networks. Moreover, the WDM technology will be increasingly used in access networks, too.

Although, in the present day there is enormous research in the area of OTDM technology (optical time division multiplex) as a parallel branch of WDM researching, it is still no possible to use OTDM technology to construct real networks. However, the extensive progress in WDM technology and its capability to produce the real components for WDM networks provide opportunities to apply WDM in the transport networks area and also in addition to other areas.

By using of WDM technology the transmission band of optical fibre functions more effectively. The experimental results have indicated that the OTDM technology will provide a more effective use of the transmission band of optical fibre. Presently, we can speak about two different technologies that will compete between each other. However, soon the both technology will be improved so they will not rival each other but probably they will cooperate together. This means that WDM technology and OTDM technology will complement each other.

If we want to employ the WDM technology into an existing infrastructure it is important to keep a certain quality of services (QoS). The operators want to use their resources so the best as it is possible, but they have to observe QoS, which will be offered to customers. The QoS is determined in terms of the blocking probability, which means the probability that any connection request will be blocked at the time t due to the insufficient resources.

The WDM network is an optical network to employ the wavelength division multiplex (WDM) technology. This technology enables transport data on different optical wavelengths through the same optical fibre. We will deal with only WDM network, where each wavelength corresponds to a data communication channel. Moreover, we will study only all-optical networks. In these networks, the optical signal still remains in the optical domain from the source node to the destination node.

An optical fibre can carry several simultaneous wavelength channels. Each wavelength has to be different on the same fibre. The number of wavelengths that each fibre can carry is limited by the physical characteristic of the fibre and the state of optical technology, which is used to combine these wavelengths onto to the fibre and isolate them off the fibre. Today the third low loss optical window (about 1550 nm) is used for the transport, which supports about tenths wavelengths. However, it is expected to grow rapidly in the next ten years [1].

The WDM transmission is transparency. This means that the wavelength channels are independent between the end nodes, provided that each wavelength channel is a data communication channel. A connection can be established between the end nodes on a WDM channel (on a wavelength). Each wavelength can transmit a data with a different transport speed, form of signal and modulation. Moreover, some of the wavelengths can transport an analogy signal (data) and other wavelengths can transport digital signal (data) through the same fibre. Another advantages of all-optical WDM networks are eliminating electronic costs and bottlenecks at intermediate nodes and that the transport speed can be increases by adding the optical wavelength. Of course, there are a few difficult requests (very steady lasers for different wavelengths and very high quality optical filter) and additional losses of WDM multiplex and demultiplex.

In the WDM networks, the path selection includes two relative individual tasks: routing and wavelength assignment problem. When we use wavelength to route data then it refers to wavelength routing. A network that employs this technique is called a wavelength routed network [2]. A wavelength routing network consists of two types of nodes. The former are the access network nodes and the second are optical cross-connectors (OXC). The OXC provides the switching and routing functions in order to establish the connection between edge nodes. The OXC can include wavelength converters, which enable a wavelength conversion. And the wavelength conversion has a significant effect on the blocking probability. Unfortunate, the wavelength converter is still a very expensive optical device today. Therefore we want to study limited wavelength conversion in node and sparse limited wavelength conversion.

If a signal is transmitted in the networks through the physical links and the signal has to use the same wavelength, then it is called transmission without wavelength conversion. The networks, which do not enable wavelength conversion, are called the networks without conversion or the non-conversion networks. This is also known as the wavelength continuity constraint. If it is possible to change a wavelength to other wavelength in the network nodes, than those networks are called networks with conversion or the conversion networks.

The wavelength conversion means the change of incoming wavelength to another wavelength. We know several types of wavelength conversion. In general, any incoming wavelength can be switched to any outgoing wavelength. The number of possible outgoing wavelengths on which the incoming wavelength can be switch is k, and the number of all outgoing wavelength is W at the node. In dependence on k, we know following type of wavelength conversion [3]:

- **no-wavelength conversion:** if the $k=1$, where each incoming wavelength is converted only to itself (any incoming wavelength is switched to the same outgoing wavelength) (Fig. 1.a),
- **fixed wavelength conversion:** if the $k=1$, where each incoming wavelength can be converted to other outgoing wavelength, but this outgoing wavelength is well-known (it is still the same) (Fig. 1.b),
- **limited wavelength conversion:** if the $1<k<W$, where each input wavelength can be converted to any wavelength from a specific limited set of wavelengths (Fig. 1.c),

- **full wavelength conversion:** if the k=W, where any incoming wavelength can be converted to any outgoing wavelength (Fig. 1.d).

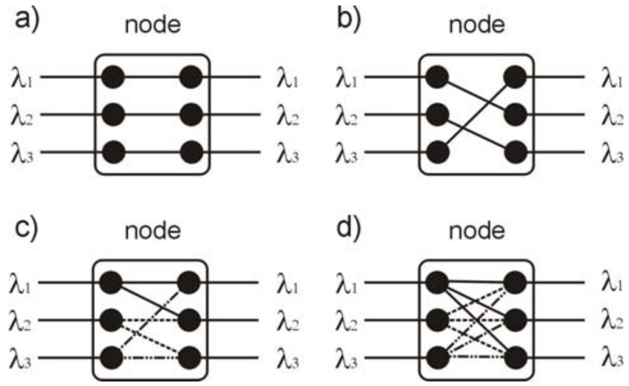

Fig.1 Types of wavelength conversion

At this point there are different possibilities how we can build up the all-optical WDM networks. We can do it without wavelength conversion, with full or limited wavelength conversion.

At present the observation focus in optical network with limited range wavelength conversion, because these networks have the performance similar to full wavelength conversion networks. However, networks with limited range conversion have lower technological demand than full conversion networks, mainly optical switches are less complicated. The all-optical networks can originate the in following ways:

A) networks with no wavelength conversion (there are only node without wavelength converter)

B) network with limited wavelength conversion:

- limited wavelength conversion due to restriction at the node (it is simply called limited wavelength conversion):
  - ⇒ all nodes of WDM network use converters only with limited wavelength conversion,
  - ⇒ all nodes of WDM network use converters only with partial wavelength conversion,
- limited wavelength conversion due to restriction in the network (it is also called sparse wavelength conversion):
  - ⇒ only a few nodes employ wavelength converters with limited wavelength conversion (other nodes employ either without or full wavelength conversion),

⇒ only a few nodes employ wavelength converters with full wavelength conversion, other nodes do not enable wavelength conversion or enable only limited wavelength conversion,

- in the case of multi-fibre WDM networks, the network does not enable wavelength, but the same wavelength can be employed on the different fibre on the same link.

C) network with full wavelength conversion (each node employs wavelength converter with full wavelength converter)

Furthermore, the network with limited wavelength conversion can originate as a combination of the limited wavelength conversion due to restriction at the node with sparse wavelength conversion due to restriction in the network. This means that the task of limited wavelength conversion is wide area.

2 Model

2.1 Review of Barry-Humblet model [4]

Barry and Humblet proposed a new model for the evaluation of an achievable utilization of network with and without wavelength conversion and for evaluation a gain wavelength conversion in [4]. This model is an analytical model, which they used to approximate the blocking probability along a path with and without wavelength conversion. This model assumes the steady state and also the link and wavelength independent assumption. There is also assumed only uniform traffic in this model. We will also suppose that each link has the same number of wavelengths F.

Let ρ be the probability that a wavelength is used on a hop. We shall use F to denote the number of wavelengths on a link and H is the number of hops. We can analyse this model as follows. First, consider networks with full wavelength conversion. Then, ρ^F is the probability, that all wavelengths are used on a hop. Hence, $1-\rho^F$ is the probability that a wavelength is a free on a hop. So, $\left(1-\rho^F\right)^H$ is the probability that a wavelength is a free on all hops along its path. And finally, we can write Barry/Humblet formula for blocking probability for networks with full wavelength conversion:

$$P_{b,full} = 1-\left(1-\rho^F\right)^H . \tag{1}$$

Now we consider a network without wavelength conversion. The probability that a wavelength is free on a hop is (1-ρ). And so, $\left(1-\rho\right)^H$ is the probability, that the same wavelength is free on all hops. Then, we can write this form $\left[1-(1-\rho)^{H}\right]$ for probability that is not such a wavelength, which is the same on all hops. Hence, we obtain Barry/Humblet form for blocking probability for networks without wavelength conversion

$$P_{b,no} = \left(1-(1-\rho)^{H}\right)^{F} . \qquad (2)$$

We can see the typical graphic functionality of blocking probability networks with full and without wavelength conversion versus the number of wavelengths in following figures (Fig. 2 and 3), respectively. In figure 2, we plot the blocking probability $P_{b,full}$ for achievable utilization ρ=0,8. Whereas, the blocking probability $P_{b,no}$ is plotted for achievable utilization ρ=0,1 in figure 3. As can be seen from figure2, the effect of path length H on blocking probability is small. While, we can see the effect of path length on blocking probability for network without wavelength conversion is significant. Note that the blocking probability is plotted for ρ=0,8 in figure 2, while it is plotted for only ρ=0,1 in figure 3. It is because the blocking probability $P_{b,no}$ dramatic increases with the increasing achievable utilization ρ in network without wavelength conversion.

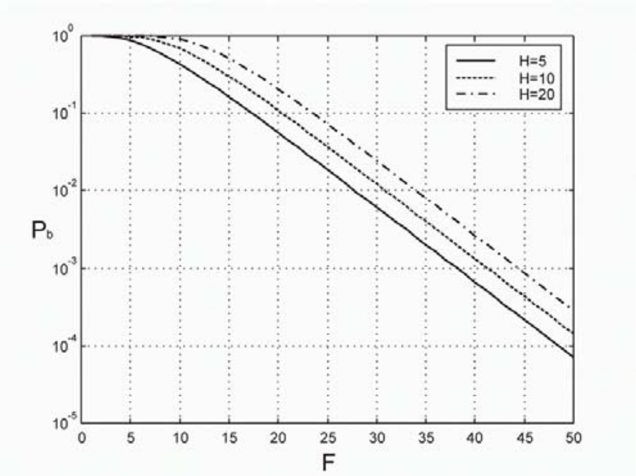

Fig.2 The blocking probability $P_{b,full}$ of the number of wavelengths for ρ=0,8

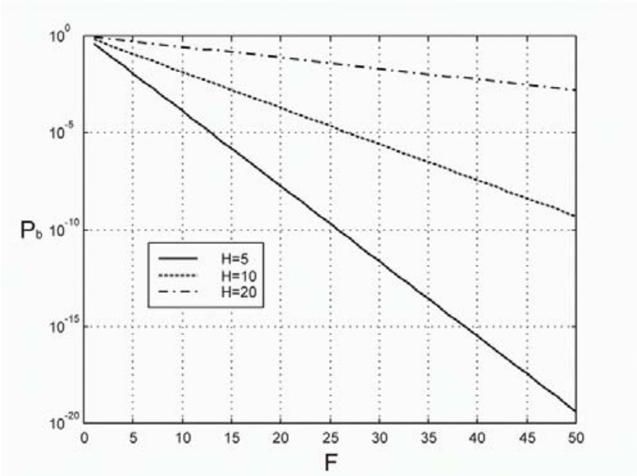

Fig.3 The blocking probability $P_{b,no}$ of the number of wavelengths for ρ=0,1

The blocking probability with full and no wavelength conversion is plotted as a function of the achievable utilization ρ in the next figures 4 and 5. The first figures (Fig. 4) is for F=10 and the second (Fig. 5) is for F=15. You can see again that the effect path length is small for networks with full wavelength conversion, but it is dramatic for networks with no wavelength conversion. The blocking probability rapidly goes near to one for the network with no wavelength conversion as the achievable utilization is increased.

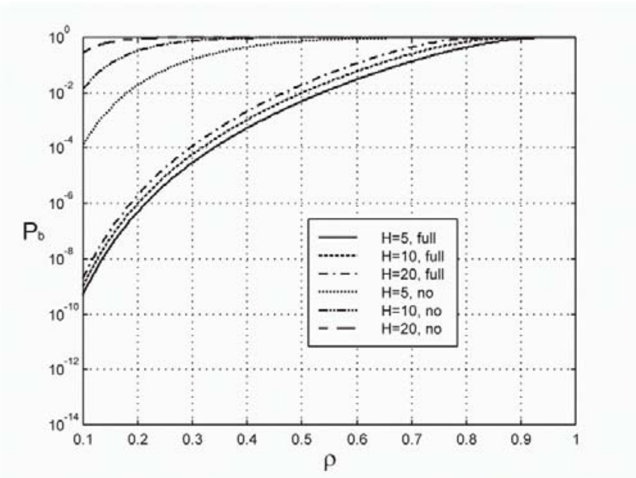

Fig.4 The blocking probability $P_{b,full}$ and $P_{b,no}$ of utilization for F=10

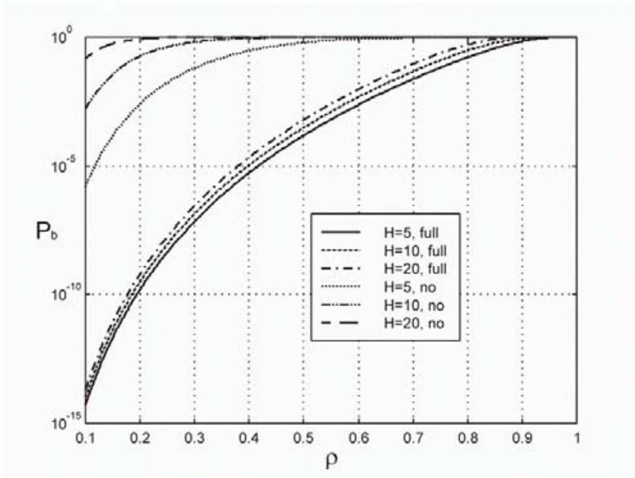

Fig.5 The blocking probability $P_{b,full}$ and $P_{b,no}$ of utilization for F=15

We shall use q denote the achievable utilization for networks with full wavelength conversion and p as the achievable utilization for networks with no wavelength conversion. We can derive formulas from Barry and Humblet form for blocking probability in networks full and no wavelength conversion, respectively:

$$q = \left[1 - \left(1 - P_{b,full}\right)^{1/H}\right]^{1/F} \tag{3}$$

$$p = 1 - \left(1 - P_{b,no}^{1/F}\right)^{1/H} . \tag{4}$$

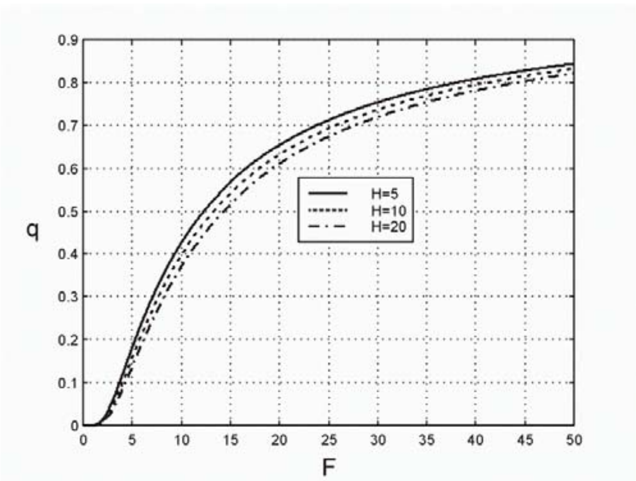

Fig.6 The q utilization of wavelength of the number of wavelengths for $P_b=10^{-3}$

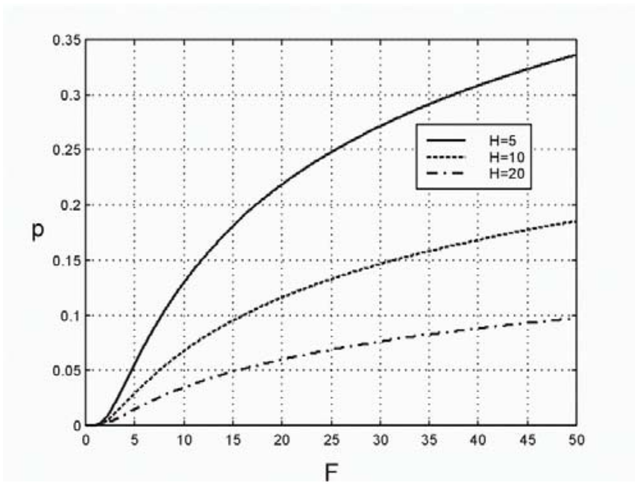

Fig.7 The p utilization of wavelength of the number of wavelengths for $P_b=10^{-3}$

Figure 6 shows the achievable utilization as the function of the number of wavelength for the networks with wavelength conversion, and the figure 7 is plotted for the networks without wavelength conversion. It is plotted for the blocking probability $P_b=10^{-3}$. Note that, the utilization is much smaller in the case of no wavelength conversion as it is in the case of full wavelength conversion for the same blocking probability.

Moreover, they have also defined the gain as a measure of benefit of the wavelength conversion. It is the rate of achievable utilization of network with full wavelength conversion q to achievable utilization of network with no wavelength conversion p, if the blocking probability is the same, it means $P_{b,no} = P_{b,full} = P_b$,

$$G = \frac{q}{p} = \frac{\left[1-\left(1-P_b\right)^{1/H}\right]^{1/F}}{1-\left(1-P_b^{1/F}\right)^{1/H}}. \qquad (5)$$

The gain as the function of the number of wavelengths is illustrated in the next figure (Fig.8) for the blocking probability $P_b=10^{-3}$. As can be seen in the figure, the effect of path length is large. At first it increases a roughly linear until a peak. Note that the peak of the gain is about H/2. After it is reach the peak, the gain is slowly decreases.

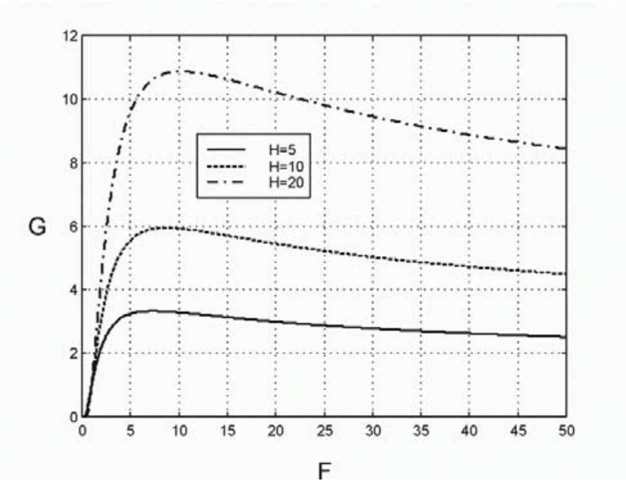

Fig.8 The gain wavelength conversion of the number of wavelengths for $P_b=10^{-3}$

2.2 The extension of Barry-Humblet model

We shall use the following notations:

- F - Number of wavelengths on each link (we assume the same number wavelengths on each link)
- H - Number of hops of path
- $l_{i,j}$ - Direct link between node i and j
- λ^R - End-to-end trafic load on path R
- $\lambda^R_{i,j}$ - Amount traffic λ^R going through link $l_{i,j}$

- $\rho_{i,j}$ - Load per wavelength over link $l_{i,j}$

A The extension of Barry-Humblet model for non-uniform traffic

Denote by λ^R end-to-end traffic load from source node to end node on path R, but it is also the arrival probability of a call request from source to destination node. We use $\lambda^R_{i,j}$ to denote the amount of traffic λ^R going thought over link $l_{i,j}$, where the link $l_{i,j}$ is direct link from node i to node j. Then, we can denote by $\rho_{i,j}$ the load per wavelength over the link $l_{i,j}$, and it can be determine by

$$\rho_{i,j} = \frac{1}{F}\sum_{\forall R}\lambda^R_{i,j} \quad . \tag{6}$$

So, we can modify the Barry/Humblet formula for the full wavelength conversion as follows

$$P_{b,full} = 1 - \prod_{\forall i,j \in R;H}\left(1-\rho^F_{i,j}\right) \tag{7}$$

and for the no wavelength conversion

$$P_{b,no} = \left(1 - \prod_{\forall i,j \in R;H}\left(1-\rho_{i,j}\right)\right)^F \quad . \tag{8}$$

B The extension of Barry-Humblet model for the limited wavelength conversion

Now we consider a network with limited wavelength conversion, where each input wavelength can be converted to any wavelength from a specific limited set of wavelengths. There are k wavelengths in each limited set of outgoing wavelengths. It means that we will assume the symmetrical limited wavelength conversion with the conversion degree d (k=2d+1), non-symmetrical limited wavelength conversion with the conversion degree d (k=d+1) „on the right side" or „on the left side" from the wavelength plane or random limited wavelength conversion [3]. Moreover, there are the same number of wavelengths on each link and each node use the same way of wavelength conversion.

In additional, we will consider that the use of wavelength on a hop is statistically independent of other hops and other wavelengths. It is good assumption for the random wavelength assignment but not for the first fit algorithm, which is the most used.

At first we consider the uniform link load this means that each link load is assumed to be the same ρ. The probability $P_{b,lim}$ that the request for a session between two pair nodes is blocked is the probability that a hop exists with all wavelengths occupied from a limited set of wavelengths k at least one of hop

$$P_{b,\lim} = \left[1-\left(1-\rho^{k}\right)^{H}\right]^{F/k}. \quad (9)$$

Note that if the k=1, which is the case of no wavelength conversion the form will be modified to $P_{b,\lim} = \left(1-\left(1-\rho\right)^{H}\right)^{F}\big|_{k=1}$. You can see that it corresponds to the formula (2). And if the k=F, which is the case of full wavelength conversion we obtain the next formula $P_{b,\lim} = 1-\left(1-\rho^{F}\right)^{H}\big|_{k=F}$, but this is nothing else but the formula for blocking probability of networks with full wavelength conversion (3).

In the next figures (Fig. 9 and 10) we want to show typical characteristic of blocking probability with limited wavelength conversion with conversion degree k=3 (d=1 for symmetrical limited wavelength conversion or d=2 for non-symmetrical limited wavelength conversion). As can be seen form figure 9 the blocking probability $P_{b,\lim}$ is less than blocking probability without wavelength conversion. But it is higher than blocking probability with wavelength conversion. In fact we have expected these results. The impact of hops is not so significant as we can see in case of no wavelength conversion from figure 10. In figure 11 is comparison of blocking probability without and limited wavelength conversion. You can see than if we just use degree of wavelength conversion k=3, the blocking probability will be decreased significantly. Note that the impact of hops is no so dramatic as you can see in the case of no wavelength conversion. It means that we can more use a wavelength on the link along a physical path for the same blocking probability.

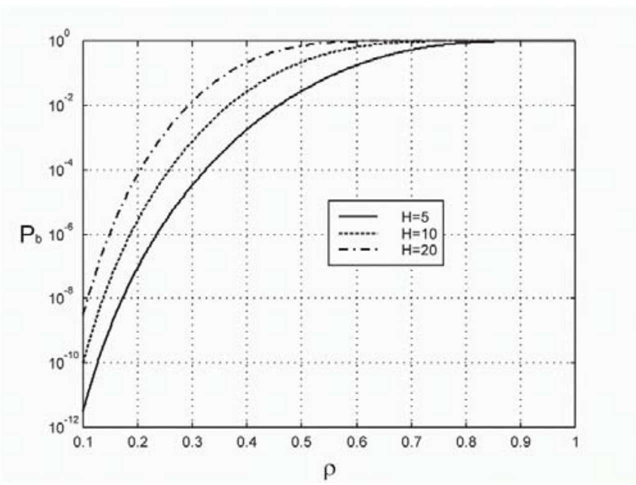

Fig.9 $P_{b,\lim}$ of wavelength utilization ρ for k=3 and F=15

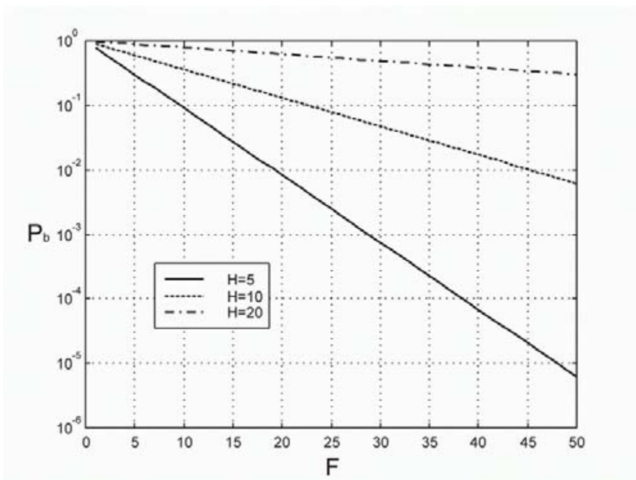

Fig.10 $P_{b,\lim}$ of the number of wavelength F for ρ=0,5 and k=3

Figure 11 shows the comparison blocking probability of networks without and limited wavelength conversion, and the next figure 12 shows the comparison blocking probability of networks full and limited wavelength conversion.

For a non-uniform link load we can modify the formula of blocking probability for the limited wavelength conversion as follows

$$P_{b,\lim} = \left[1 - \prod_{i,j \in R;H} \left(1 - \rho_{i,j}^{k} \right) \right]^{F/k} . \qquad (10)$$

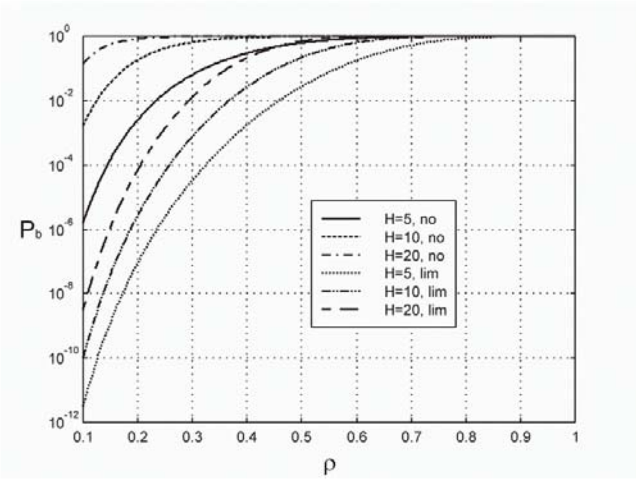

Fig.11 $P_{b,lim}$ and $P_{b,no}$ of wavelength utilization ρ for k=3 and F=15

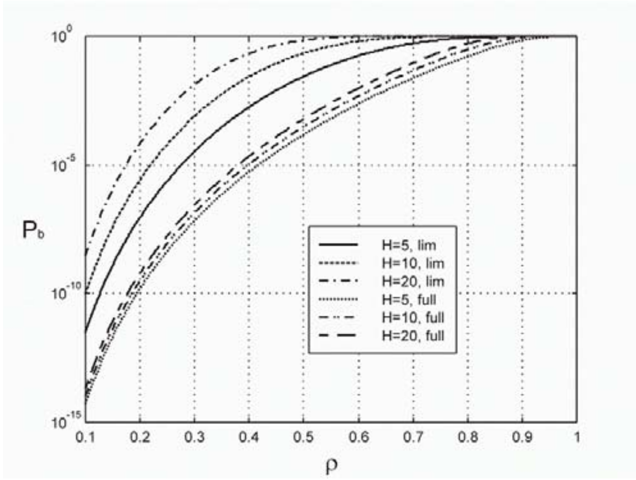

Fig.12 $P_{b,lim}$ and $P_{b,full}$ of wavelength utilization ρ for k=3 and F=15

3 The using of Barry-Humblet model for evaluation blocking probability in the networks with a sparse wavelength conversion

We first implement Barry-Humblet model to determine the end-to-end blocking probability between any pair of nodes in the network with sparse wavelength conversion, where only a few nodes employ wavelength converter with full wavelength conversion. And then we will deal with the effect limited wavelength conversion and we will determine performance of limited wavelength conversion in sparse wavelength conversion.

We shall assume that the route between source node and destination node is known to be path R. We shall consider only a single path case. We define a segment of the path as a set of hops between two directly consecutive nodes with any converter or between an end node and its nearest converter node in the path [5, 6] (Fig. 13).

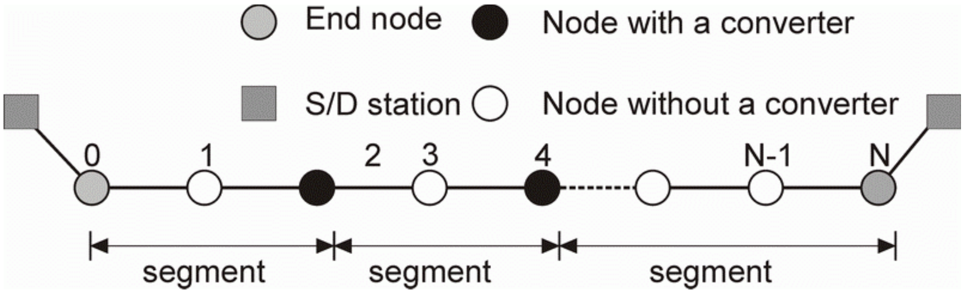

Fig. 13 Definition of segment

We shall use the following notations:

- N - Number of nodes
- n - Number of converters
- s - Number of segments, if n=0 then s=1 else s=n+1;
- ℓ_j - Diameter of j-th segment, j=1,...,s

3.1 Sparse full-no wavelength conversion

In the case of sparse full-no wavelength conversion we assume that there are only a few nodes with full wavelength conversion in the network (therefore ''full''). Other nodes are without wavelength conversion (hence ''sparse''). We will consider the end-to-end blocking probability between any pair of end nodes.

The blocking probability P_b^R between end pair nodes along the route R can be by computed by following expression

$$P_b^R = 1 - \prod_{j=1}^{s} \left\{1 - P_{b,no}^{seg_j}\right\} \tag{11}$$

where $P_{b,no}^{seg_j}$ is the blocking probability of segment j. Note that the blocking probability each segment is formulated as blocking probability without wavelength conversion, because there are no wavelengths within a segment. So, we can expressed P_b^R as

$$P_b^R = 1 - \prod_{j=1}^{s} \left\{1 - \left[1 - (1-\rho)^{\ell_j}\right]^F\right\} \tag{12}$$

and s is the number of segments and ℓ_j is the diameter of j-th segment. The diameter of segment is the number of its nodes. Of course, the above-mentioned formula is for uniform traffic load on each link. For non-uniform load traffic we have the following formula

$$P_b^R = 1 - \prod_{j=1}^{s} \left\{1 - \left[1 - \prod_{i=1}^{\ell_j} \left(1 - \rho^{j,i}\right)\right]^F\right\} \tag{13}$$

where $\rho^{j,i}$ is link load of the i-th link from j-th segment.

3.2 Sparse full-limited wavelength conversion

There are only a few nodes with the full wavelength conversion and all other nodes employ a wavelength converter with the limited wavelength. The blocking probability is given by

$$P_b^R = 1 - \prod_{j=1}^{s} \left\{ 1 - P_{b,\lim}^{seg_j} \right\} \tag{14}$$

where $P_{b,no}^{seg_j}$ is the blocking probability of segment j. We can calculate this probability by next formula

$$P_{b,\lim} = \left[1 - \left(1 - \rho^k \right)^{\ell_j} \right]^{F/k} \tag{15}$$

where d is conversion degree. For non-uniform traffic we can use instead of term (15) formula (10).

4 Numeric result

In this section we give numeric results of the extended Barry Humblet model. We applied the extended Barry-Humblet model for an evaluation of the blocking probability for the single path case with 10 hops (H=10) and 11 nodes (N=11). The first we calculated the blocking probability for a uniform link load, and we assumed the link load per wavelength ρ is 0,5. There were 15 wavelengths on each link.

In table 1 we can see the numeric results of the blocking probability for the sparse full-no and full-limited wavelength conversion as a function of converter place. Note that in this case the link load is the same on each hop. There are only the minimum and maximum of blocking probability for the number of converter with full wavelength conversion n. You can see that sometimes there are several possibilities where can be situate converters. For example there are 35 possibilities how can be put six wavelength converters in the network and the blocking probability will be still the same. Hence, we have a lot of option where the wavelength converter can be given in the network. And we can optimalize the placement of the wavelength converters.

TABLE 1 The blocking probability as a function of converter place for F=15, H=10, for a uniform load ρ=0,5 and for a limited wavelength conversion is k=3						
	Min.			Max.		
	Full-No	**Full-Limited**		**Full-No**	**Full-Limited**	
n	$P_{b,\,min}$	$P_{b,\,min}$	**Place**	$P_{b,\,max}$	$P_{b,\,max}$	**Place**
0				0.98545	0.21732	00000 00000
1	0.85645	0.05409	00001 00000	0.98545	0.21732	00000 00001
2	0.53589	0.01986	00010 01000 00100 01000 00100 10000	0.97110	0.16731	00000 00011 10000 00001
3	0.27153	0.00922	00100 10100 00101 01000 01001 01000 01010 01000 00101 00100 01001 00100	0.94299	0.12190	00000 00111 10000 00011 11000 00001
4	0.06506	0.00353	01010 10100	0.88902	0.08269	00000 01111 10000 00111 11000 00011 11100 00001
5	0.05245	0.00289	15 ; example : 01010 10110	0.78963	0.05100	5 ; example : 00000 11111
6	0.03967	0.00224	35 ; example : 01111 01010	0.62118	0.02757	6 ; example : 00001 11111
7	0.02673	0.00160	28 ; example : 01111 10110	0.37993	0.01232	7 ; example : 11100 01111
8	0.01360	0.00095	9 ; example : 01111 11110	0.13512	0.00413	8 ; example : 11110 01111
9	0.00031	0.00031	11111 11110	0.01360	0.00095	9 ; example 01111 11111
10	0.00031	0.00031	11111 11111			

For example, we can also see the blocking probability is from 0.27153 to 0.94299 for the case of there is 3 converters in the network in dependence on the place of converters in the case of full-no sparse wavelength conversion. And it is from 0.00922 to 0.12190 for full-limited sparse wavelength conversion. The range of blocking probability as a function of converter place is really widely for the certain number of the wavelength converters. Therefore it is important to find the optimal place for it. Of course from this example we can see that the blocking probability is still high for 3 converters in the full-no conversion network. It is because we set very high ρ (the probability that a wavelength is used on a hop ρ is 0,5 for each hop) and the number of hops is quite a lot of (H=10). For ρ=0,3 the blocking probability is only 0.00375 for the optimal placement of the converters and it is 0.41039 for the worst placement (the maximum of the blocking probability).

When we use the node with limited conversion instead the nodes with no wavelength conversion the blocking probability will be dramatic degrease for the same placement of the converters with the full wavelength conversion.

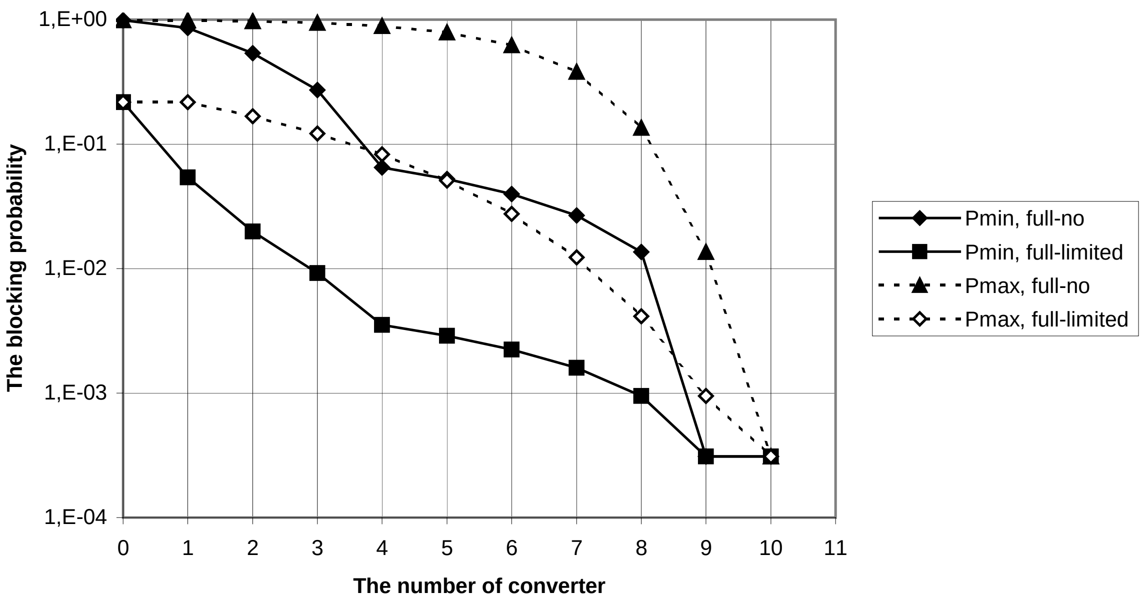

Fig. 14 The blocking probability as a function of place of converter and the number of converter

The figure 14 shows the blocking probability $P_{b,min}$ for the best place of converters and $P_{b,max}$ for the worst place as a function of the number of converters. From this figure we can also see that the blocking probability dramatic decrease when we use 2, 3 and 4 wavelength converters. It is clearly that it is not necessary to use more then 4 wavelengths converter in the networks with 10 hops. And it is also not effectively.

Our network is very simply it is only a path which include 10 hops. We should apply the extended Barry-Humble model for a mesh network. But it is out of the scope of this paper. But if we want to compute the overall blocking probability of the network we can divide this network into the paths of the hops and we can determine the blocking probability of the path. For the overall blocking probability we can use the following formula [5]

$$B = \frac{\sum_R \lambda_R P_b^R}{\sum_R \lambda_R}. \tag{16}$$

The table 2 and figure 15 show also result for the simple path networks with 10 hops. And there are 15 wavelengths on each hop. But the link load is a non-uniform. In this case ρ_i is a vector for i=1,2,...,H, and we set ρ=[.2 .8 .5 .3 .4 .3 .6 .2 .5 .4].

TABLE 2
The blocking probability as a function of converter place for F=15, H=10, for a non-uniform load, and for a limited wavelength conversion at k=3

ρ=[.2 .8 .5 .3 .4 .3 .6 .2 .5 .4];

	Min.				Max.			
	Full-No		**Full-Limited**		**Full-No**		**Full-Limited**	
n	$P_{b, min}$	**Place**	$P_{b, min}$	**Place**	$P_{b, max}$	**Place**	$P_{b, max}$	**Place**
0					0.966661	00000 00000	0.255082	00000 00000
1	0.733435	00010 00000	0.068543	01000 00000	0.966661	00000 00001	0.255082	00000 00001
2	0.343678	01000 10000	0.043170	01000 10000	0.958490	10000 00001	0.251867	10000 00001
3	0.121230	01001 01000	0.037773	01000 11000	0.931717	10000 00011	0.225687	10000 00011
4	0.082077	01010 10100	0.036413	11000 11000	0.867809	00000 01111 10000 00111	0.175080	10000 00111
5	0.044482	11010 10100	0.035910	11100 11000	0.837412	10000 01111	0.172156	10000 01111
6	0.037870	11010 11010	0.035726	11100 11010	0.639333	10000 11111	0.093271	10000 11111
7	0.036394	11101 11010	0.035713	11101 11010	0.525629	10001 11111	0.085670	10001 11111
8	0.035971	11101 11110 11110 11110	0.035703	11101 11110	0.337034	10011 11111	0.068707	10011 11111
9	0.035699	11111 11110	0.035699	11111 11110	0.206290	10111 11111	0.062241	10111 11111
10	0.035699	11111 11111	0.035699	11111 11111				

There is also the minimum of the blocking probability for the optimal place of the wavelength converters in the left column and the maximum of the blocking probability for the worst place of it in the right column. We can again see that there are very high differences between the minimum and maximum blocking probability in dependence on place of wavelength converters and also on the number of wavelength converters. It means that it is possible to vary the place for the converters and to find the best place for them in terms of the low blocking probability. Note that sometimes the optimal place for converters in the full-no sparse wavelength conversion is different as the optimal place for them in the case of full-limited wavelength conversion.

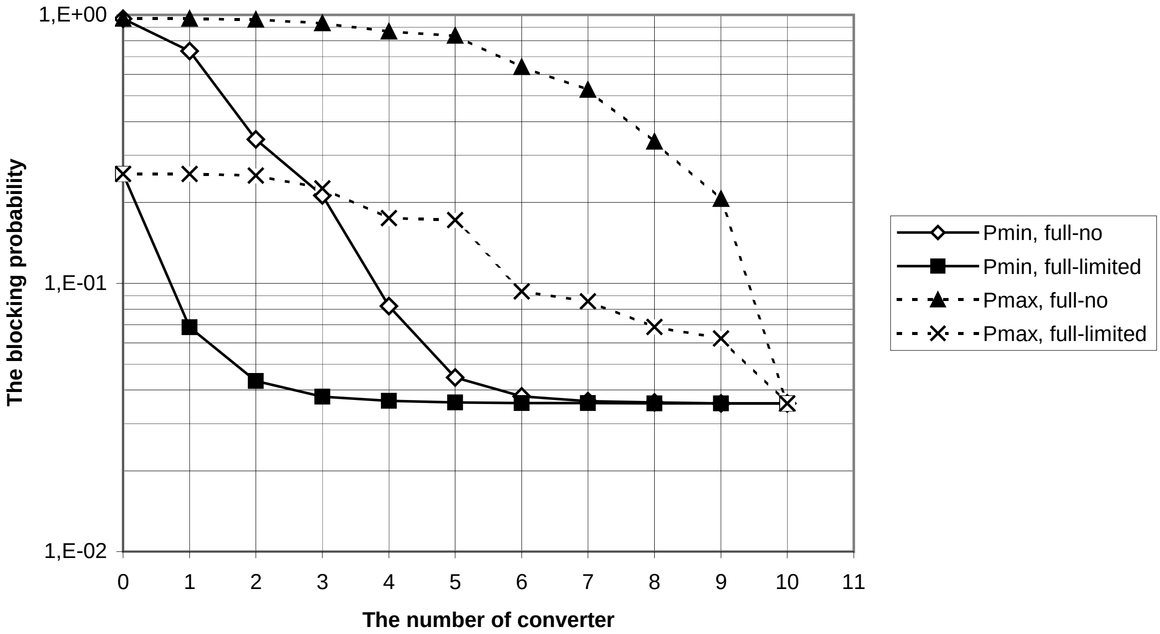

Fig. 15 The blocking probability as a function of place of converter and the number of converter

From the figure 15 we can again see that the effective number of wavelength converters is at most four. For it, the difference between the minimum and maximum blocking probability is dramatic, and it is really necessary to determine the optimal placement for the converters.

5 Conclusion

This paper gives the extension of analytical Barry-Humblet model for an evaluation of the blocking probability for a non-uniform link load for the network with full and without wavelength conversion. Also, we have extended the Barry-Humblet model for the network with the limited wavelength conversion. The converter with the full wavelength conversion is still very expensive today. Therefore the problem of limited wavelength conversion is high actually.

Our model gives very interesting information about the behaviour a network with limited wavelength conversion. We demonstrate that the blocking probability of the network with limited wavelength conversion is much lover than the blocking probability of the network without wavelength conversion for not very high conversion degree. Also, the impact of the number of hops is no such dramatic as in the network with no wavelengths conversion. Due to the limited wavelength conversion we can increase the link load ρ for the certain blocking probability. It also means the diameter of networks can be higher. The performance of the limited wavelength conversion for small conversion degree is very good. It is not very effective to set up the very high conversion degree. In our case it was k=3, it means that d=1 for the symmetrical conversion and d=2 for the non-symmetrical case of the limited wavelength conversion.

The extended Barry-Humble model was applied to determine the optimal place for wavelength converter in the network with sparse conversion. This model was used for two kinds of the networks with the sparse wavelength conversion. The former is full-no sparse wavelength conversion and the second is full-limited sparse conversion. But unfortunate it was very simply networks include only 10 consecutive hops and 11 nodes. However, we demonstrated that it is very important to find the optimal placement of wavelength converter. Therefore we can see that the range between the blocking probability for the optimal placement and for the worst place is really significant.

Also we can say that if the wavelength converters are placed optimal it is not necessary to used a lot of its. Note that, in our case it is effective to use at most 4 wavelength converters.

In future we want to explore the case of a sparse limited conversion with limited-no conversion. There will be only node with limited wavelength conversion and without wavelength conversion in the network. And also we will try to implement our extended Barry-Humblet model for mesh networks.

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