THE BLOCKING PROBABILITY MODEL IN ALL-OPTICAL NETWORKS FOR LIMITED WAVELENGTH CONVERSION

Miroslav Bahleda
Department of Telecommunication,
Faculty of Electrical Engineering,
University of Zilina
Slovakia
bahleda@fel.utc.sk

Miroslav Bystrianky
Department of Telecommunication,
Faculty of Electrical Engineering,
University of Zilina
Slovakia
bystriansky@fel.utc.sk

Karol Blunar
Department of Telecommunication,
Faculty of Electrical Engineering,
University of Zilina
Slovakia
blunar@fel.utc.sk

ABSTRACT
Optical WDM networks are providing the huge bandwidth and they are still very attractive in backbone networks. In this paper we study the blocking probability model by Barry and Humblet for single fiber WDM networks and model by Al-Zahrani et al. for multifiber WDM networks. The model by Barry and Humblet is proposed for no and full wavelength conversion and we extend this model for limited wavelength conversion. In fact, the proposed model is generalized for no, limited and full wavelength conversion. The results of these models are compared for single fiber and multifiber networks. Due to the results, taking into account the wavelength utilization and the blocking probability the optimal network could be designed with respect to costs of components.

KEY WORDS
Optical WDM networks, blocking probability model by Barry and Humblet and model by Al-Zahrani et al., new blocking probability model for limited wavelength conversion

1. Introduction

Optical networks, which employ the wavelength division multiplex (WDM) technology, are called WDM networks and they are still a popular architectural solution for the core networks of wide area networks. WDM technology provides transmission on different optical wavelengths through the same optical fiber.
In a WDM network, it is possible to route data to their respective destinations based on their wavelengths and it is referred to as *wavelength routing*. A network which employs this technique is known as a *wavelength routed* network [1]. Such a network consists of two types of nodes:

- Optical cross-connects (OXC), which connect the fibers in the networks,
- End nodes (access or edge nodes), which provide the interface between non-optical end systems and the optical systems.

In wavelength routed networks, in order to transfer data from the source node to the end node, a wavelength continuous route has to be set up at the optical layer, which is called a *lightpath*. It is assumed that the optical signal still remains in the optical domain between two access nodes. Such a network is called *all-optical network* (*photonic network*). All lightpaths using the same fiber link must allocate different and distinct wavelengths. It is also known as *distinct wavelength assignment constraint* [1].
We will deal with optical WDM networks only, where each different wavelength correspondents to a data communication channel. It is known as a *circuit switched optical WDM network*.
The OXC provides the switching and routing functions in order to establish the connection between edge nodes. The OXC can include wavelength converters (WCs) for supporting wavelength conversion. A wavelength converter is a device which is able to switch any incoming optical signal at its port from a wavelength to different wavelength among the available wavelengths from its output port.
If the nodes have the optical cross-connects without wavelength converters, the same wavelength must be used on all the fiber links along physical path for establishing a lightpath. This is also known as *wavelength continuity constraint* [1]. Such a network is called network with no wavelength conversion. A connection request is accepted only if there is at least one wavelength which is simultaneously free on all the links of that route. It means a call can be blocked even if there are free wavelengths on all the links, but they are not the same one.
However, if the nodes employ wavelength converters, different wavelengths can be assigned for a lightpath on each fiber link along the physical path. A connection request is accepted if on all the links on its route there is at least one free wavelength. However, the converters are still very expensive and there are also technological limitations. The implementation of all-optical full wavelength conversion is quite difficult and it is also not

economically feasible to place wavelength converters to all network nodes. Therefore, we deal with limited wavelength conversion, which is described later in this paper.

The problem of blocking probability in wavelength-routed optical WDM networks have been studied previously in [2]-[5]. Barry and Humblet proposed an analytical model in [3] to determine the end-to-end blocking probability in all-optical networks with and without wavelength conversion. We improve the model by Barry and Humblet for networks with limited wavelength conversion. Models for the multifiber networks have been proposed by Al-Zahrani in [2]. Therefore a multifiber wavelength routed networks with F fibers per link and W wavelengths per fiber is equivalent to a single-fiber networks with $F.W$ wavelengths with the nodes having conversion degree F. We compare our model with this scenario.

The rest of paper is structured as follows. The basic networks and traffic assumption and review of model by Barry and Humblet and model by Al-Zahrani are described in section II. We explain our model for limited wavelength conversion in section III. Numerical results and comparison are presented in section IV and conclude in section V.

2. The basic networks and traffic assumption and review of model by Barry-Humblet and model by Al-Zahrani

The analytical model by Barry and Humblet was proposed to determine the end-to-end blocking probability in all-optical networks with and without wavelength conversion. In this model, there are considered two independence assumptions:

- Link independence assumption: the link states on different links are independent.
- Wavelength independence assumption: the individual wavelengths are utilized independently of the utilizations of other wavelengths on the same link.

The both link and wavelength independence assumptions lead to the overestimation of the blocking probability. The analysis of blocking probability is simpler due to these factors. However, it means that this model is inaccurate, but it has a moderate computational complexity. Moreover, the model by Barry and Humblet is used to approximate the blocking probability only along a path (consecutive links) with full and no wavelength conversion.

In this paper, the following parameters and notations are used:

W	Number of wavelengths on each link per fiber
H	Number of hops along a path
F	Number of fibers per hop
Ch	Number of channel in a hop; $Ch=W.F$
$l_{i,j}$	Direct link between node i and j
a	End-to-end traffic load on a path
$a_{i,j}$	Amount of traffic a going through link $l_{i,j}$
$\rho_{i,j}$	Load per wavelength over link $l_{i,j}$

In this model, it is assumed the call request is arrived on each link as Poisson process with rate λ and the connection holding time is exponentially distributed with mean μ. Then the load is expressed as $a=\lambda/\mu$. The wavelengths utilization (load per wavelength) ρ is the probability that a wavelength is used on a link and it can be computed by

$$\rho_{ij} = \frac{\sum_R a_{ij}}{W} \qquad (1)$$

where R is link path.

It is assumed the same link load over all links, i.e. $\rho_{ij}=\rho$ (uniform link load) and the same number of wavelengths on each link. The blocking probability at time t is the probability that at least one connection request (lightpath) will be blocked before time t.

2.1 Model by Barry and Humblet [3]

For networks without wavelength conversion, the blocking probability $P_{b,no}$ between any pair of nodes is the probability that each wavelength is used on at least one of the H hops. It is expressed as

$$P_{b,no} = \left[1-(1-\rho)^H\right]^W \qquad (2)$$

It is derived as follows. The load per wavelength ρ is the probability that a wavelength is used on a hop. Hence, the probability that a wavelength is free on a hop is $(1-\rho)$. And $(1-\rho)^H$ is the probability that the same wavelength is free on all hops. Then, we can write $[1-(1-\rho)^H)]$ for the probability that there is not the same free wavelength on all hops. Hence, the model by Barry and Humblet is expressed as (2) for W wavelengths.

In networks with full wavelength conversion, the blocking probability $P_{b,full}$ is the probability that there is a hop with all wavelengths used and it is given by

$$P_{b,full} = 1-\left(1-\rho^W\right)^H \qquad (3)$$

It is derived as follows. Note again that the load per wavelength ρ is the probability that a wavelength is used on a hop. Then, ρ^W is the expected probability that all wavelengths are used on a hop. The probability that a wavelength is a free on a hop is $1-\rho^W$. Hence, the probability that a wavelength is free on all hops along its path is $(1-\rho^W)^H$. Then, the model by Barry and Humblet is expressed as (3).

2.2 Model by Al-Zahrani et al. [2]

The model by Al-Zahrani et al. was proposed to determine the blocking probability between the source-destination pair in all-optical multifiber networks with and without wavelength conversion. In a multifiber networks, a link hop between two intermediate nodes consists of a bundle of optical fibers. Because the number of wavelengths that each fiber can carry is limited by the physical characteristic of the fiber and the state of optical technology, an alternative approach for increasing the number of channels is to light multiple fibers. Each fiber provides the same set of wavelengths.

In this model, the assumptions and notations are the same as above-mentioned in this section. The blocking probability in multifiber networks without wavelength conversion is the probability that each wavelength in every fiber is used on at least one of the *H* intermediate hops. In other words, the new lightpath request is blocked on a wavelength, if this wavelength is not free on all of the *F* fibers on a hop along the path. It is thus given by

$$P'_{b,no} = \left[1-\left(1-\rho^{F}\right)^{H}\right]^{W} \qquad (4)$$

The blocking probability in multifiber networks with full wavelength conversion is the probability that there are all wavelengths used in every fiber on at least one of the hops along the path. It is expressed as

$$P'_{b,full} = \left[1-\left(1-\rho^{F.W}\right)^{H}\right]^{W} \qquad (5)$$

where *F.W* is the total number of channels *Ch* in a hop. These formulas are derived from (2), (3) by taking into consideration the multifiber scenario and the detail explain is outside the scope of this paper.

2.3 Results of models

In Fig. 1-2 the blocking probability of networks with and without wavelength conversion is plotted as a function of wavelength utilization ρ for H=5, 10, 20. Fig. 1 and 2 show the blocking probability for the single and multi fiber networks, respectively. In this scenario, multifiber network consists of 3 fibers per hop with 4 wavelengths per fiber and there are 12 wavelengths for single fiber network. It means the total number of channel is the same for both single fiber and multifiber networks.

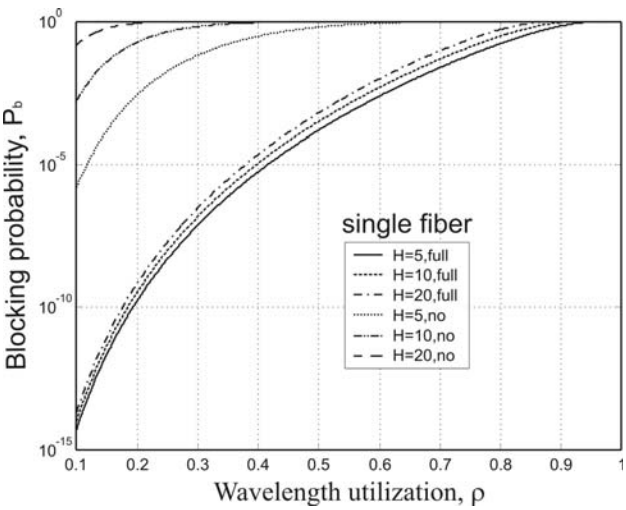

Figure 1. Blocking probability as a function of wavelength utilization for single fiber no and full wavelength conversion network for *W*=15, *H*=5, 10 and 20.

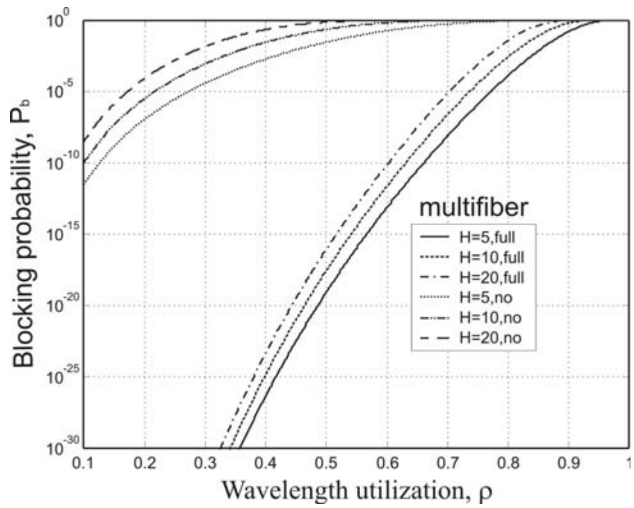

Figure 2. Blocking probability as a function of wavelength utilization for multifiber no and full wavelength conversion network for *W*=5, *F*=3 and *H*=5, 10 and 20.

From these figures we can obtain the interesting results. Unfortunate, the detail explanations are out of scope of this paper. In general, the blocking probability increases with wavelength utilization and the number of hops. The blocking probability of networks without wavelength conversion depends on the number of hops significantly unlike the networks with wavelength conversion. Fig. 3 shows in detail that the multifiber network with *F* fibers and *W* wavelengths per fiber achieves better performance than the single fiber network with *F.W* wavelengths per fiber.

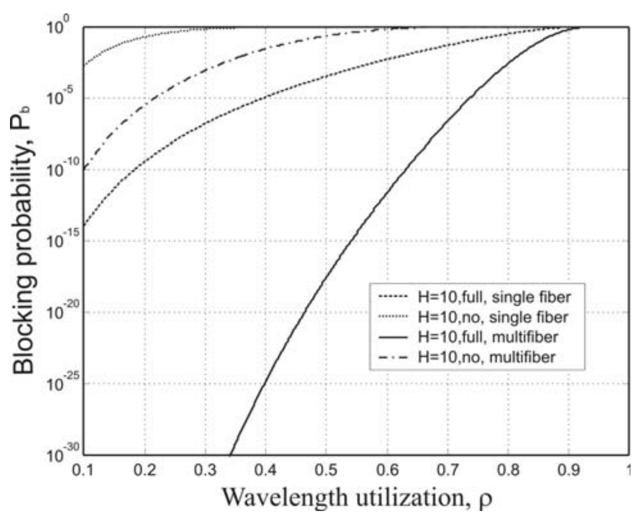

Figure 3. Blocking probability as a function of wavelength utilization for single fiber network with *W*=15 and multifiber network with *W*=5 and *F*=3. The number of hops is 10. Note that there is the same number of total channels per link.

In Fig. 4 the blocking probability is plotted for single fiber full wavelength conversion networks in compare with multifiber no wavelength conversion networks. However, the number of wavelengths is the same on each fiber for the both single fiber and multifiber network in contrast to previous cases. Note that for the small wavelength utilization the blocking probability of multifiber no wavelength conversion networks is lower then for single fiber full wavelength conversion networks, in contrast to the large wavelength utilization.

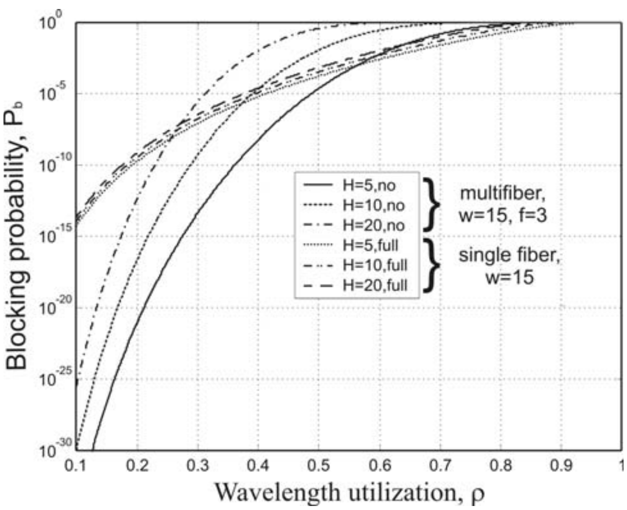

Figure 4. Blocking probability as a function of wavelength utilization for single fiber network with W=15 and multifiber network with W=15 and F=3. Note that there is the same number of wavelengths per link.

3. Model for limited wavelength conversion

Since the all optical wavelength converters with full range conversion are still very expensive due to technological limitations it leads to the focus on networks with limited wavelength conversion [6]. In the limited wavelength conversion networks it is allowed conversion but with some restrictions. They can be of following types [7, 8]:

- A limited range of wavelengths to which an input wavelength may be converted (it is referred to as *limited wavelength conversion with conversion degree k*) (Fig. 6).
- A limited number of converters at the nodes of the network (it is referred to as *sparse wavelength conversion*) (Fig. 5).
- A limited number of wavelength converters are placed at the node (it is referred to as *partial wavelength conversion*).

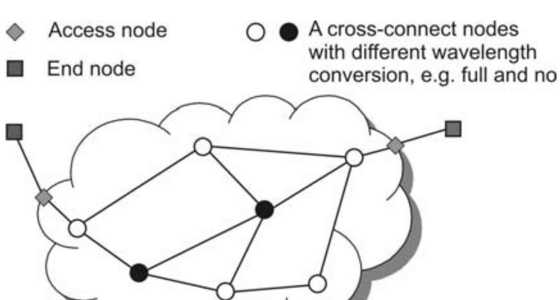

Figure 5. Network with sparse wavelength conversion

In general, the limited wavelength conversion with limited range of wavelength conversion with conversion degree k means that any incoming wavelength can be converted to k outgoing wavelengths on output side from wavelength plane of the spectrum [7]. The wavelength plan is an order of all outgoing wavelengths. In practice, the following types of limited wavelength conversion are used:

- *Symmetrical*: Any incoming wavelength can be converted to d adjacent outgoing wavelengths on left and right side from the wavelength plane of the spectrum, as well as the same wavelength. This means that any incoming wavelength can be converted to k=2d+1 outgoing wavelengths (Fig. 6).
- *Non-symmetrical*: Any incoming wavelength can be converted to the same wavelength or to one on the left (or the right) side from the wavelength plane. It means that it is possible to switch any ingoing wavelength to k=d+1 outgoing wavelengths (Fig. 6).

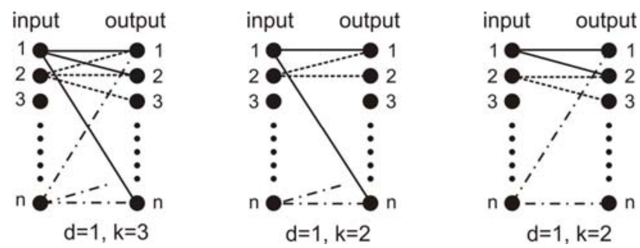

Figure 6. Symmetrical limited wavelength conversion with conversion degree k=3 (d=1) and non-symmetrical limited wavelength conversion with conversion degree k=2 (d=1)

Using above-mentioned notations and assumption we introduce our model. This model is proposed for the networks with limited wavelength conversion with a conversion degree k. It is derived from model by Barry and Humblet taking into consideration that the each incoming wavelength can be converted to one wavelength from k outgoing wavelengths. The probability $P_{b,lim}$ is the probability that there is at least one hop with all occupied wavelengths from a limited set of wavelengths k

$$P_{b,\lim} = \left[1-\left(1-\rho^k\right)^H\right]^{W/k} \qquad (6)$$

It is derived as follows. Note that ρ is the probability that a wavelength is used on a hop. Then ρ^k expresses the expected probability that k wavelengths, on which can be converted any incoming wavelength, are occupied on a hop. The probability that a suitable wavelength is a free on a hop is 1-ρ^k. Hence, the probability that a suitable wavelength is free on all hops along its path is $(1-\rho^k)^H$. The factor W/k represents the effect of limited wavelength conversion with conversion degree k.

Note that when no wavelength conversion is considered then k equals 1 and the expression (6) can be modified to (2). And for full wavelength conversion the conversion degree k is equal to W and the expression (6) can be changed to (3).

Moreover, a multifiber no wavelength conversion network with F fibers per link and W wavelengths per fiber is equivalent to a single fiber limited wavelength conversion network with *F.W* wavelengths and conversion degree *F*. The equivalence can be derived from (6) by setting $k \to F$ and $W \to WF$.

Hence, our model (6) is generalized form to determinate the blocking probability for single fiber networks with no (k=1), full (k=F) and limited ($1<k<F$) wavelength conversion and also for multifiber no wavelength conversion network with k fibers and W/k wavelengths per fiber.

4. Numerical results

In this section, we present the numeric results for our model and we compare the performance of single fiber and multifiber networks with different wavelength conversions. For single fiber and multifiber networks we have used model by Barry and Humblet and model by Al-Zahrani, respectively. The results for the single fiber network with limited conversion are computed by our model, which is presented in previous section.

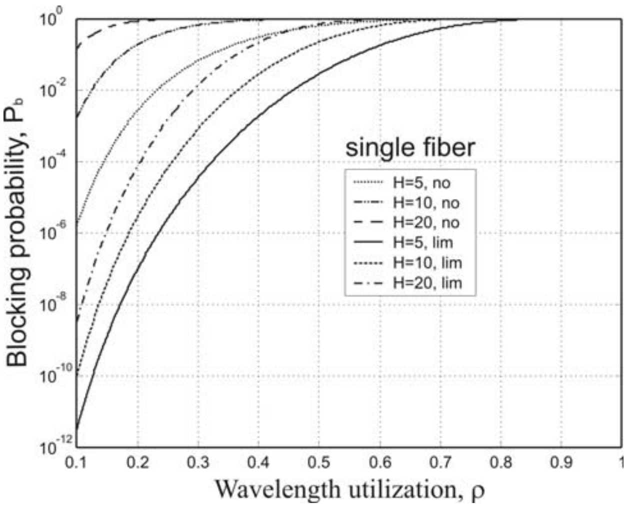

Figure 7. Blocking probability as a function of wavelength utilization for single fiber network with no and limited wavelength conversion for W=15 and H=5, 10 and 20.

In Fig. 7-9 we have plotted the results of blocking probability as a function of wavelength utilization for W=15 and H=5, 10 and 20. In generally, each figure shows that blocking probability increases with the number of hops H and always increases along the wavelength utilization ρ. Note that, wavelength conversion reduces the blocking probability and thus increases wavelength utilization.

In Fig. 7 there can be seen the comparison of the blocking probability for single fiber networks with no and limited wavelength conversion. Note that the figure is for conversion degree k=3. We can see that the improvement due to the use of limited wavelength conversion is considerable with respect to the performance of the no wavelength conversion. Thanks to limited wavelength conversion it is able to keep the required blocking probability together with higher wavelength utilization or higher number of hops as shows the figure.

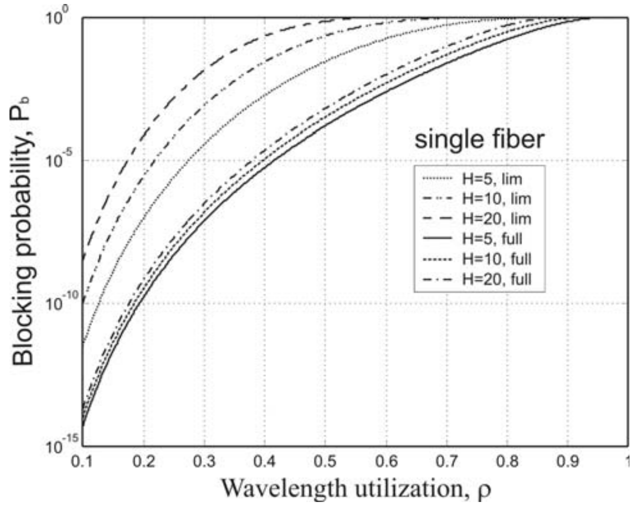

Figure 8. Blocking probability as a function of wavelength utilization for single fiber network with limited and full wavelength conversion for W=15 and H=5, 10 and 20.

Fig. 8 shows the blocking probability for single fiber networks with full and limited wavelength conversion. We can see that the performance obtained by limited wavelength conversion with degree k=3 is very similar to the performance of full wavelength conversion and for the large conversion degree k, it is almost close to the performance of full wavelength conversion. Note that the effect of the number of hops for limited wavelength conversion is not as dramatic as it is for no wavelength conversion but it is still significant with consider to full wavelength conversion.

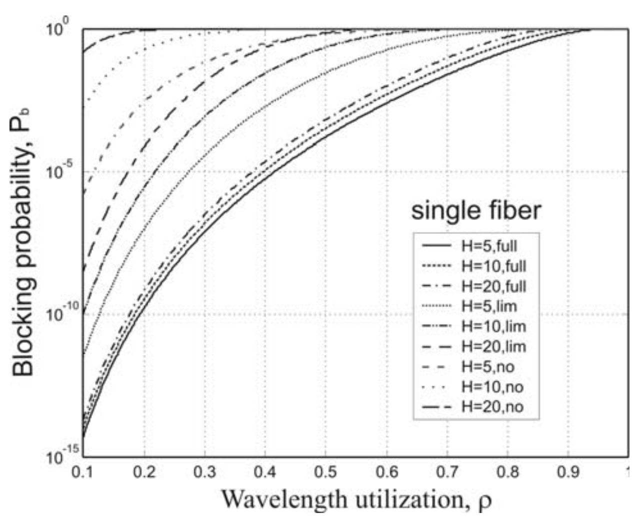

Figure 9. Blocking probability as a function of wavelength utilization for single fiber network with no, limited and full wavelength conversion for W=15 and H=5, 10, 20. Note that the results of limited wavelength conversion in single fiber network are the same for multifiber no wavelength conversion network with k fibers and W/k wavelengths per fiber.

Fig. 9 is plotted for the single fiber no, limited and full wavelength networks to compare the effect of wavelength conversion. The network capacity can be improved by increasing the number of wavelengths per fiber or the number of the fibers per hop or by adding the wavelength converters to an existing network. We suppose that the number of wavelengths is still limited by technological improvements. Moreover, the multifiber full wavelength networks are economically ineffective, definitely. Hence, we do not deal with multifiber full wavelength conversion case.

The tree curves from this figure, which are plotted for single limited wavelength conversion with the number of wavelength W and conversion degree k, are the same for the multifiber no wavelength conversion network with $F=k$ fibers and W/k wavelengths per each fiber. Therefore, based on the component costs (fibers or wavelength converter) and the blocking probability (Fig. 9) an optimal solution can be determined to improve the network capacity by taking into account the wavelength utilization.

5. Conclusion

In this paper we presented our model for single fiber network with limited wavelength conversion. This form is generalized for the blocking probability for single fiber networks with no (k=1), full ($k=F$) and limited ($1<k<F$) wavelength conversion and also for multifiber no wavelength conversion network with k fibers and W/k wavelengths per fiber.

The results of this model are compared to the model by Barry and Humblet for single fiber case and by Al-Zahrani for multifiber case. Although these models are not very accurate due to the link and wavelength independence assumptions, they give a very interesting result, which is presented in this paper. The blocking probability as a function of wavelength utilization for the certain number of wavelengths and given number of hops, fibers per hop or conversion degree is also presented here. Hence, taking into account the wavelength utilization and the blocking probability it could be designed the optimal networks with respect to costs of components.

In this paper we assume uniform link load due to the simplicity. However, it should be noted that the results of this paper can be extended for the non-uniform link load.

The model by Barry and Humblet is also used to determinate the optimal placement of wavelength converters for sparse wavelength networks in [5]. In such networks, some network nodes have a wavelength converter, but not all nodes. The blocking probability depends on the converter placement significantly. Hence, the optimization problem is to decide which network nodes should be equipped with the converter in order to minimize the overall network blocking probability. In [5] the nodes are equipped with the converters enabling the full wavelength conversion, only. In future, we want to extend it for limited wavelength conversion and implement our model. The research in this area leads to determinate the type of wavelength conversion, the number wavelengths and fibers in order to design optimal networks with respect to customer and operator demands. In future, we will simulate different cases of wavelength conversion in VPI simulation tools and compare the simulation results with the numeric results. Afterwards, based on these results we will optimize today's networks.

6. Acknowledgements

This work was supported by Science and Technology Assistance Agency under the contract No. APVT-20-022404, Technologies for all-optical processing for next generation digital optical networks.

References:

[1] C. S. R. Murthy, M. Gurusamy, *WDM optical networks: concepts, design and algorithms* (Prentice Hall PTR, New Jersey, 2002).

[2] F. A. Al-Zahrani, A. A. Habiballa and A. P. Jayasumana, Path Blocking Performance in Multi-Fiber Wavelength Routing Networks with and without Wavelength Conversion: *Proc. 12th International Conference on Computer Communications and Networks*, Dallas, TX, 2003, 580-583.

[3] R. A. Barry, P. A. Humblet, Models of Blocking Probability in All-Optical Networks with and Without Wavelength Changers, *IEEE Journal on Selected Areas in Communication*, *14*, (5), June 1996, 858-867.

[4] A. Birman, Computing Approximate Blocking Probabilities for a Class of All-Optical Networks, *IEEE Journal on Selected Areas in Communications*, *14* (5), June 1996, 852-857.

[5] S. Gao, X. Jia, C. Huang, and D.-Z. Du, An Optimization Model for Placement of Wavelength Converters to Minimize Blocking Probability in WDM Networks, *Journal of Lightwave technology*, *21*(3), March 2003, 684-694.

[6] R. Odrobinak, M. Dado, Different wavelength conversion technologies using in optical cross-connect: *Proc. 7th International Conference on Transparent Optical Networks,* Barcelona 2005, Spain, pp. 417-420.

[7] Bahleda, M., Blunar, K., The wavelength conversion in WDM networks, *Komunikácie – the scientific letters of University of Zilina*, *6*(4), EDIS, Žilina 2004, 98-102.

[8] B. Ramamurthy, B. Mukherjee, Wavelength Conversion in WDM Networking, *Journal of Selected Areas in Communication*, *16*, September 1996, pp. 1061-1073.